Benha University

Faculty of Engineering- Shoubra

Eng. Mathematics & Physics Department

Preparatory Year

(تخلفات)



Final Term Exam

Date: December 26, 2018 Course: Mathematics 1 – A

Duration: 3 hours

The Exam consists of one page Answer All Questions No. of questions: 4 Total Mark: 100

Question 1

(a)Complete the following:

5

3

7

- (i)The eigenvalues of a symmetric matrix of real numbers are
- (ii) A square matrix A is called singular if and it is called symmetric if
- (iii) If λ is eigenvalue of a matrix A, then 3^{λ} is eigenvalue to
- (iv)A linear system AX = B is called homogenous if
- (v)A linear system AX = B has infinite number of solutions if

(b) If
$$A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix}$$
, $B = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -3 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$

Find, if possible A + B, A + C, A.B, A.C, C.A, |A|, |C.A|

(c) Find the eigenvalues and the eigenvectors of the matrix : $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$

Also, Write the Hamilton equation, the diagonal form and find $f(A) = 3^A$.

Question 2

(a) Solve the L.S: (i)
$$3x - y + z = 3$$
, $2x + 2z = 6$, $-x + y + z = 2$

(ii)
$$x - y + 2z = 2$$
, $-2x - z = -3$, $3x - y + z = 3$

(b) Find
$$S_n$$
 and S_{10} from the series : $\sum_{r=1}^n \frac{1}{r^2 + 3r + 2}$

(c) By Induction, prove that :
$$1 + 2 + 3 + \dots + n = \frac{1}{2}n(n+1)$$

(d)If
$$z_1 = 1 - 2i$$
, $z_2 = 2 + 5i$. Find $z_1 + z_2$, $z_1 \cdot z_2$, $(z_1 + z_2)^9$ and $\sqrt{z_1 + z_2}$.

Question 3

- (a) For what values of x the function $f(x) = \frac{e^{\sin x}}{4 \sqrt{x^2 9}}$ continuous.
- (b) Use the limit definition to find the first derivative for: $f(x) = 5x^2 3x + 7$.

(c) Find y` where : (i)
$$y = (3x^2 + 1)^{\frac{1}{x}} + \frac{(\ln x)^x}{2^{3x^5 - 1}}$$
 (ii) $y = \frac{x \csc x}{3 - \csc x} + x^2 \cdot \sec^2 \pi x$

Question 4

(a) Find the limits: (i)
$$\lim_{x\to 0} \frac{3^x - 2^x}{x^2 - x}$$
 (ii) $\lim_{x\to 0+} (\tan x)^{x^2}$

- (b) Write the Maclaurin's expansion of the function : $f(x) = x^2 \cos x$.
- (c) If the sum of two numbers is constant k. Find the minimum sum of their squares.

(d) Find the integrals : (i)
$$\int \frac{1 + \ln x}{5 + x \ln x} dx$$
 (ii) $\int \frac{1}{x^3} (7 + \frac{5}{x})^{-3} dx$